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Ques:— Definition and examples of Metric spaces.
 Ans:—

Definition: Metric space;— let X be a non empty set. A function d of $X \times X \rightarrow \mathbb{R}$ (the set of reals) such that $d: X \times X \rightarrow \mathbb{R}$ is called a metric or distance function iff d satisfies the following axioms

$$[m1]: d(x, y) \geq 0 \text{ for all } x, y \in X$$

$$[m2]: d(x, y) = 0 \text{ iff } x = y$$

$$[m3]: d(x, y) = d(y, x) \text{ for all } x, y \in X \text{ (Symmetry)}$$

$$[m4]: d(x, y) \leq d(x, z) + d(z, y) \text{ for all } x, y, z \in X$$

(triangle inequality)

If d is a metric for X then the pair (X, d) is called a metric space and d is called the distance between x and y

The axiom [m1] states that the distance between any two points of X is a non negative real number.

Axiom [m2] states that if the two points coincide then the distance between them is zero and if the distance between the two points is zero, the two points are the same

Axiom [m3] states that the distance between two points x and y does not depend on the order of the given points x and y

Axiom [m4] known as the triangle inequality states figuratively that the sum of lengths of two sides of a triangle is greater than or equal to the length of the third side. In fact the sign of equality occurs when these points are in a straight line.

Note:— If $x \neq y$ then [m1] and [m2] $\Rightarrow d(x, y) > 0$
 so that the distance between two distinct points is positive.

